A Plastic-Damage Model for Concrete under Cyclic Loads

Atila Sarikaya[1], Emre Erkmen[2], Nadarajah Gowripalan[3], Vute Sirivivatnanon[4] and Warren South[5]
[1]PhD Student, University of Technology Sydney, NSW, Australia
[2]Assistant Professor, Concordia University, Canada
[3]Senior Research Fellow, University of Technology Sydney, NSW, Australia
[4]Professor of Concrete Engineering, University of Technology Sydney, NSW, Australia
[5]Director, Research and Technical Services, Cement Concrete & Aggregates Australia

Abstract: A constitutive model based on a novel coupled elastoplastic-damage framework is adopted for the modelling of concrete under cyclic loads. Coupled elastoplastic-damage models have been used to capture both the material degradation and the permanent deformations under inelastic deformations. In this study, a multisurface plasticity framework is implemented for the modelling of concrete under compressive and tensile cyclic loads. The elastoplastic-damage framework is based on the 'direct-coupling' method in which an a-priori relationship between the total strain and the damage strain is postulated. The model is easy to calibrate since it utilises the same yield and potential functions for plasticity and damage calculations. Concrete is modelled using a pair of yield surfaces in order to capture its compressive and tensile behaviour while utilising corresponding isotropic damage variables to capture the stiffness degradations in the compressive and tensile regimes. Material parameters are calibrated using uniaxially loaded concrete experiments. The results are compared with experimental and numerical data provided in the literature.

Keywords: coupled elastoplastic-damage, strong coupling, multisurface plasticity, cyclic load, stiffness degradation.

1. Introduction

Coupled elastoplastic-damage constitutive models have been successfully utilised for prediction of inelastic behaviour of materials, including concrete. By virtue of the plasticity and damage components of the constitutive model, both the irreversible deformations and the stiffness degradation can be captured. When subjected to excessive forces, concrete exhibits stiffness degradation as well as permanent deformations subsequent to unloading. These characteristics of concrete lead to development of coupled plastic-damage models by various researchers including Ortiz (1), Lubliner, Oliver et al. (2), Oller, Oñate et al. (3), Abu-Lebdeh and Voyiadjis (4), Meschke, Lackner et al. (5), Lee and Fenves (6), Yazdani and Schreyer (7, 8), Grassl and Jirásek (9), Jason, Huerta et al. (10), Wu, Li et al. (11), Cicekli, Voyiadjis et al. (12), Červenka and Papanikolaou (13), Voyiadjis, Taqieddin et al. (14, 15), Al-Rub and Kim (16), Grassl, Xenos et al. (17), among many others.

While the additive decomposition of the strain tensor into elastic and plastic parts is well established within the plasticity framework, treatment of damage has been more diverse. For instance, physical characterisation of damage mechanisms (e.g., isotropic damage or localised cracks) can be quite different among the damage models in the literature. Likewise, while some damage models consider an explicit 'damage strain' term, others do not. See Krajcinovic (18), and Armero and Oller (19) for a comprehensive perspective.

Earliest attempts for coupling plasticity and damage through decomposing the total strain into elastic, plastic and damage components can be found in Klisiński and Mróz (20), Yazdani and Schreyer (7). Later, Abu-Lebdeh and Voyiadjis (4), Lubarda and Krajcinovic (21), Armero and Oller (19), Hansen, Willam et al. (22), Al-Rub and Voyiadjis (23), Ibrahimbegović, Markovič et al. (24), Brünig (25), and Ibrahimbegović, Jehel et al. (26) also decomposed the strain tensor by taking into account a damage strain component.

By utilising damage surfaces defined in the stress space and employing the damage strain concept, Armero and Oller (19) introduced a novel framework in which the treatment of different damage models was unified. In their approach, the sharing of plastic and damage parts in the total strain is determined based on the equilibrium condition of updated stresses between the plastic and damage components. This approach was successfully adopted for the analysis of a wide variety of materials, including metals and concrete (Ibrahimbegović, Markovič et al. (24), Ibrahimbegović, Jehel et al. (26), Ayhan, Jehel et al. (27)). It is also notable that while the damage strain is reversible, the damage evolution, which is characterised by a separate damage variable, is irreversible in the framework of Armero and Oller (19).

Based on Armero and Oller (19)'s method, Sarikaya and Erkmen (28) introduced a novel 'direct-coupling' framework in which an a-priori relationship between the total strain and the damage strain was proposed in terms of the damage variable. Their strong coupling approach eliminates the necessity of iterations to satisfy the equilibrium condition of updated stresses between the plastic and damage components.

In this paper, the direct coupling framework in Sarikaya and Erkmen (28) is adopted in order to investigate the inelastic behaviour of concrete under full-cyclic loads. The concrete constitutive model in the latter reference is extended to capture the tensile behaviour of concrete. A pair of damage variables are employed in order to characterise different damage mechanisms activated by tensile and compressive stresses. The model behaviour is compared with experimental and numerical results from the literature.

The outline of this paper is as follows: In the next section, we review the directly-coupled elastoplastic-damage model outlined in Sarikaya and Erkmen (28), and extend it to a multi-surface framework to cover both tensile and compressive characteristics of concrete in a single model. Section 3 is allocated to validation examples, and we conclude in Section 4.

2. Coupled Elastoplastic-Damage Model

2.1 Background: Direct-coupling method of Sarikaya and Erkmen (28)

In this paper, we will follow the analytical and computational framework provided in detail in Sarikaya and Erkmen (28). In their framework, the coupled plasticity and damage equations are built considering three basic hypotheses as summarised below:

a) The additive decomposition of the total strain tensor:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \boldsymbol{\varepsilon}_p + \boldsymbol{\varepsilon}_d \tag{1}$$

where elastic ($\boldsymbol{\varepsilon}_e$), plastic ($\boldsymbol{\varepsilon}_p$) and damage ($\boldsymbol{\varepsilon}_d$) strains constitutes the total strain tensor, $\boldsymbol{\varepsilon}$.

b) Stored strain energy (Ψ):

$$\Psi(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_p, \kappa_p, \phi, \boldsymbol{\varepsilon}_d, \kappa_d) = \Psi^e(\boldsymbol{\varepsilon}_e) + \Psi^d(\boldsymbol{\varepsilon}_d, \phi) + \Xi^p(\kappa_p) + \Xi^d(\kappa_d) \tag{2}$$

where $\Psi^e$and Ψ^d denote elastic and damage strain energies, respectively. It should be noted that the damage strain energy is also a function of the damage parameter ϕ, which indicates the level of isotropic damage. The damage parameter can be expressed as $\phi = \varphi/(1-\varphi)$ in terms of the reduction factor $\varphi \in [0,1]$, which represents the reduction in the load carrying area. Hardening effects are accounted by plastic hardening (κ_p) and damage hardening (κ_d) parameters, which, are the only variables of the plastic and damage hardening potentials denoted by Ξ^p and Ξ^d, respectively.

c) Elastic domain:

$$\Phi_p(\boldsymbol{\sigma}, q_p) = \Phi_d(\boldsymbol{\sigma}, q_d) \leq 0 \tag{3}$$

where $\boldsymbol{\sigma}$ is the stress tensor while Φ_p and Φ_d denote plastic and damage failure criteria for compression, respectively. These criteria, defined in stress space, determine the domain in which no evolution of internal variables, i.e., $\boldsymbol{\varepsilon}_p, \kappa_p, \phi, \boldsymbol{\varepsilon}_d, \kappa_d$, takes place. It should be noted that, while the equity of plasticity and damage failure criteria in Eq. (3) is an essential assumption of the direct-coupling framework of Sarikaya and Erkmen (28) which we follow here, they are usually considered independent in other models in the literature. The stress-like hardening variables for plasticity and damage are defined as $q_p = -\frac{\partial \Xi^p}{\partial \kappa_p}$ and $q_d = -\frac{\partial \Xi^d}{\partial \kappa_d}$, respectively.

Considering the thermodynamics principle that the total inelastic dissipation is always non-negative, the dissipation inequality can be obtained from Eq. (3) in the form of $\mathrm{d}\Omega = \boldsymbol{\sigma}^{\mathrm{T}}\mathrm{d}\boldsymbol{\varepsilon} - \mathrm{d}\Psi \geq 0$, where $\mathrm{d}\Omega$ is the total dissipation. By employing the classical associated plasticity approach, one can obtain an associated flow rule by maximisation of the dissipation. However, non-associated flow rule is adopted for concrete in compressive stress regime which can be expressed as in Eqs. (4) and (5) for plasticity and damage components, respectively:

$$\mathrm{d}\boldsymbol{\varepsilon}_p = \mathrm{d}\lambda_p \mathbf{b} \tag{4}$$

$$\boldsymbol{\sigma}\mathrm{d}\phi = \mathrm{d}\lambda_d \mathbf{E}\mathbf{b} \tag{5}$$

where $\mathbf{b} = \frac{\partial \Theta_p(\boldsymbol{\sigma}, q_p)}{\partial \boldsymbol{\sigma}}$ is the direction of the plastic strain increment, $\Theta_p(\boldsymbol{\sigma}, q_p)$ is the potential function for plasticity, $\mathbf{E}$ is the elastic constitutive matrix whereas $\mathrm{d}\lambda_p$ and $\mathrm{d}\lambda_d$ are the plastic and damage component proportionality factors, respectively. Note that equity of plastic and damage potentials are also assumed here. By applying the loading/unloading conditions, proportionality factors and stress increments can be determined independently for both plasticity and damage components (Ibrahimbegović, Markovič et al. (24)). Using the stress equity of both components (Armero and Oller (19)), plasticity and damage components can be coupled and the stress-strain relationship can be obtained as shown in Eq. (6) (Sarikaya and Erkmen (28)):

$$\mathrm{d}\boldsymbol{\sigma} = \mathbf{K}^{ed}[\mathbf{K}^{ed} + \mathbf{C}^{ep}]^{-1}\mathbf{C}^{ep}\mathrm{d}\boldsymbol{\varepsilon} \tag{6}$$

where $\mathbf{C}^{ep} = \mathbf{E} - \frac{\mathbf{E}\mathbf{b}\mathbf{a}^{\mathrm{T}}\mathbf{E}}{\mathbf{a}^{\mathrm{T}}\mathbf{E}\mathbf{b} - \frac{\partial \Phi_p}{\partial q_p}\frac{\partial q_p}{\partial \kappa_p}\frac{\partial \kappa_p}{\partial \lambda_p}}$ is the elastoplastic tangent modulus and $\mathbf{a} = \frac{\partial \Phi_p}{\partial \boldsymbol{\sigma}}$.

In a recent study, the first two authors of this paper proposed a 'direct coupling' relationship between the increments of total strain and damage strain (Sarikaya and Erkmen (28))

$$\mathrm{d}\boldsymbol{\varepsilon}_d = \frac{\phi}{(1+\phi)}\mathrm{d}\boldsymbol{\varepsilon} \tag{7}$$

which, with the other assumptions outlined thereof, yielded to a simplified yet effective framework in which all the variables related to damage calculations could be determined in terms of plasticity variables and damage variable ϕ, i.e.,

$$\mathbf{K}^{ed} = \phi^{-1}\mathbf{C}^{ep} \tag{8}$$

and

$$\mathrm{d}\lambda_d = \phi\mathrm{d}\lambda_p \tag{9}$$

In the same study, the authors also showed that in their framework there was no need for performing damage calculations in parallel to plasticity since they always yield to the same update of stress in the computational algorithm.

2.2 Improvement: Extension of the model to capture the tensile behaviour

In this paper, as an improvement to the compression regime model explained above, we introduce an additional criterion to capture the tensile behaviour of concrete:

$$\Phi_p^t(\boldsymbol{\sigma}, q_s^t) = \Phi_d^t \leq 0 \tag{10}$$

in which q_s^t is the tensile softening function whereas Φ_p^t and Φ_d^t are the plastic and damage failure criteria for tension, respectively. Also, we introduce a tensile damage variable ϕ^t by which the stiffness degradation in the tensile regime will be determined.

Employing the relationships given in Eqs. (4) to (9) by substituting the tensile variables with the compressive ones defined in Section 2.1, we obtain for the tensile regime

$$\mathrm{d}\lambda_d^t = \phi^t \mathrm{d}\lambda_p^t \tag{11}$$

$$\mathrm{d}\boldsymbol{\sigma} = \mathbf{K}^{t^{ed}}\left[\mathbf{K}^{t^{ed}} + \mathbf{C}^{ep}\right]^{-1}\mathbf{C}^{ep}\mathrm{d}\boldsymbol{\varepsilon} \tag{12}$$

$$\mathbf{K}^{t^{ed}} = \phi^{t^{-1}}\mathbf{C}^{ep} \tag{13}$$

where $\mathrm{d}\lambda_p^t$ and $\mathrm{d}\lambda_p^t$ are the plastic and damage Lagrange multipliers, respectively. $\mathbf{K}^{t^{ed}}$ is the damage tangent modulus for the tensile regime.

2.3 Combination of compression and tension models

Starting with the compression model and with the addition of the tensile model, we have the ingredients of the combined model, which should be capable of capturing both the compressive and tensile behaviours of the material. However, a criterion is required to combine these two independent models. Here we employ a simple criterion:

If $\Phi_p(\boldsymbol{\sigma}, q_p) = 0$ → Use compressive regime formulation (Section 2.1), or,

if $\Phi_p(\boldsymbol{\sigma}, q_p) < 0$ and $\Phi_p^t(\boldsymbol{\sigma}, q_s^t) = 0$ → Use tensile regime formulation (Section 2.2).

It should be noted that a more sophisticated criterion which combines compressive and tensile components can be employed, for instance, that of Červenka and Papanikolaou (13) or Feenstra and de Borst (29).

2.4 Specifics adopted for the concrete tension model

In addition to the compression regime model presented in Sarikaya and Erkmen (28), here we adopt a modified Rankine criterion by which the tensile behaviour of concrete can be captured. The modified Rankine criterion can be expressed in the unified Haigh-Westergaard stress space as

$$\Phi_p^t(\xi, \rho, \theta, \kappa_p^t) = \xi + \frac{1}{\sqrt{2}}\rho\, r(\theta, e^t) - \sqrt{3}\frac{f_t}{f_c} q_s^t(\kappa_p^t) \leq 0 \tag{14}$$

where, ξ, ρ and θ are the three coordinates of Haigh-Westergaard space and $r(\theta, e^t)$ is the elliptic function whereas e^t is the eccentricity, which determines the out-of-roundness of the deviatoric trace of the Rankine surface. We have kept the e^t parameter in Eq. (14) to gain control on the sharpness of edges of the Rankine yield surface and to avoid numerical convergence problems accordingly. f_t and f_c represent the uniaxial tensile and compressive strength of concrete, respectively. Note that the last term in Eq. (14) includes a function for tensile softening law, $q_s^t(\kappa_p^t)$, which is adopted from Feenstra and de Borst (29):

$$q_s^t(\kappa_p^t) = \exp\left(-\frac{\kappa_p^t}{n_3}\right) \tag{15}$$

where n_3 is a material parameter which controls the steepness of the softening part of the uniaxial tension stress-strain curve and κ_p^t is the tensile softening variable which is defined as a special uncoupled case of hardening/softening parameters in Feenstra and de Borst (29):

$$\mathrm{d}\kappa_p^t = \mathrm{d}\lambda_p^t \tag{16}$$

where $\mathrm{d}\lambda_p^t$ can be obtained from the flow rule and corresponding Kuhn-Tucker conditions of associated plasticity:

$$\mathrm{d}\lambda_p^t = \frac{\left(\frac{\partial \Phi_p^t}{\partial \boldsymbol{\sigma}}\right)^{\mathrm{T}} \mathbf{E}}{\left(\frac{\partial \Phi_p^t}{\partial \boldsymbol{\sigma}}\right)^{\mathrm{T}} \mathbf{E} \left(\frac{\partial \Phi_p^t}{\partial \boldsymbol{\sigma}}\right) - \frac{\partial \Phi_p^t}{\partial q_s^t}\frac{\partial q_s^t}{\partial \kappa_p^t}} \tag{17}$$

For the evolution of tensile stiffness degradation, we introduce the tensile damage reduction factor which varies exponentially with respect to the softening parameter:

$$\varphi^t = \frac{\phi^t}{1+\phi^t} = 1 - \exp\left(-C^t\, \kappa_p^t\right) \tag{18}$$

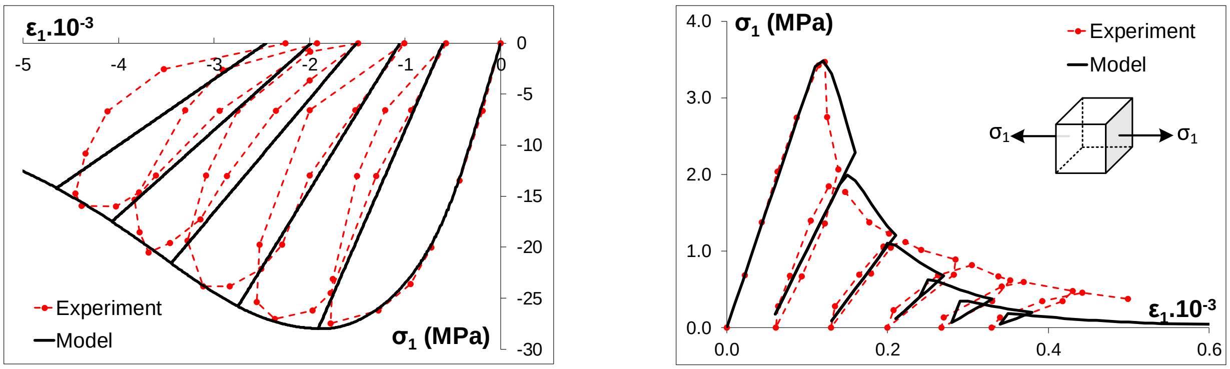
where C^t is a parameter which can be calibrated using the cyclic tension experimental data.

3. Numerical Examples

In this section, comparisons of numerical analysis results and experimental data are presented in order to illustrate the applicability of the proposed model. The parameters of the developed model are calibrated by adjusting the uniaxial compressive strength f_c and its corresponding total strain ε_c, uniaxial tensile strength f_t, Young's modulus E_c, Poisson ratio ν, the stress at the onset of plastic flow k_o, tensile softening parameter n_3, eccentricity e^t and the damage parameters for a) compression (C) and b) tension (C^t).

Comparisons presented here are limited to uniaxial loading only. Nevertheless, the model can be employed for multiaxial load cases. In the graphs, σ_1 is the axial stress and ε_1 is the axial strain.

In Figure 1, compression-only and tension-only loading results for plain concrete are presented. The compression loading results given in (a) are the same as given in Sarikaya and Erkmen (28) since no modification is made on the compression model. The tension loading results in (b) are calibrated by adjusting the parameters as E_c=31,000 MPa, ν=0.2, f_c=28 MPa, f_t=3.48 MPa, n_3=0.0025, e^t=0.5 and C^t=285. Other parameters are not shown since they have no effect on the uniaxial tensile behaviour. It can be verified that the stiffness degradation in both compression and tension cases are captured very accurately.

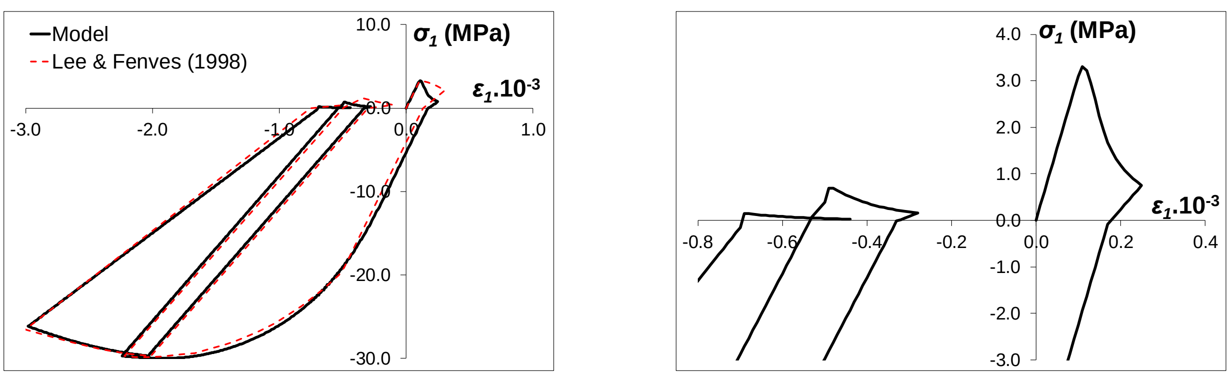


(a) Comparison with Karsan and Jirsa (30)

(b) Comparison with Gopalaratnam and Shah (31)

Figure 1. Comparison of cyclic behaviour of concrete: (a) compression-only, (b) tension-only

In Figure 2, the numerical results of a concrete element subjected to cyclic loads are presented. In (a), the results are compared with those of Lee and Fenves (6) whereas in (b), the tensile region of the graph in (a) is enlarged. The parameters are set as E_c=31,000 MPa, ν=0.2, f_c=30 MPa, k_o=0.14, ε_c=-0.00205, C=0.43, f_t=3.3 MPa, n_3=0.0025, e^t=0.5 and C^t=285. The proposed model can capture the compression and tension stiffness degradations, which evolve independently from each other.



(a) Comparison with Lee and Fenves (6)

(b) Enlarged view of the model response about the tensile region

Figure 2. Numerical results of loading with multi-cycles: (a) Comparison with Lee and Fenves (6), (b) Enlarged view of the tensile region in (a).

4. Conclusions

The direct-coupling method of Sarikaya and Erkmen (28) that was developed concrete under compression is extended to capture the inelastic tensile behaviour of concrete under cyclic loading. A simple multi-surface plasticity-damage framework is employed for that purpose. The model captures permanent deformations as well as stiffness degradations induced by independent damage mechanisms of compression and tension regimes. Effects of the loading history can be reflected by the model by preserving the hardening/softening and damage variables of the compressive and tension regimes separately. Partial and full cyclic loading examples showed that the model can provide good agreement with the experimental and numerical results in the literature.

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